Province of the
EASTERN CAPE
EDUCATION

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GRADE 12 *GRAAD 12*	

NATIONAL/*NASIONALE* SENIOR CERTIFICATE/*SERTIFIKAAT*

GRADE 12/*GRAAD 12*

SEPTEMBER 2022

MATHEMATICS P2/*WISKUNDE V2* SPECIAL ANSWER BOOK/*SPESIALE ANTWOORDEBOEK*

Marker/*Merker*				Moderator's Initials / *Moderator se paraaf*																
Question *Vraag*	Mark *Punt*			Initial Parafeer	Marks *Punte*			S M	Marks *Punte*			Đ M	Marks *Punte*			P M	Marks *Punte*			NM
1																				
2																				
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7																				
8																				
9																				
10																				
TOTAL *TOTAAL*																				

This special answer book consists of 18 pages./
Hierdie spesiale antwoordeboek bestaan uit 18 bladsye.

QUESTION 1/*VRAAG 1*

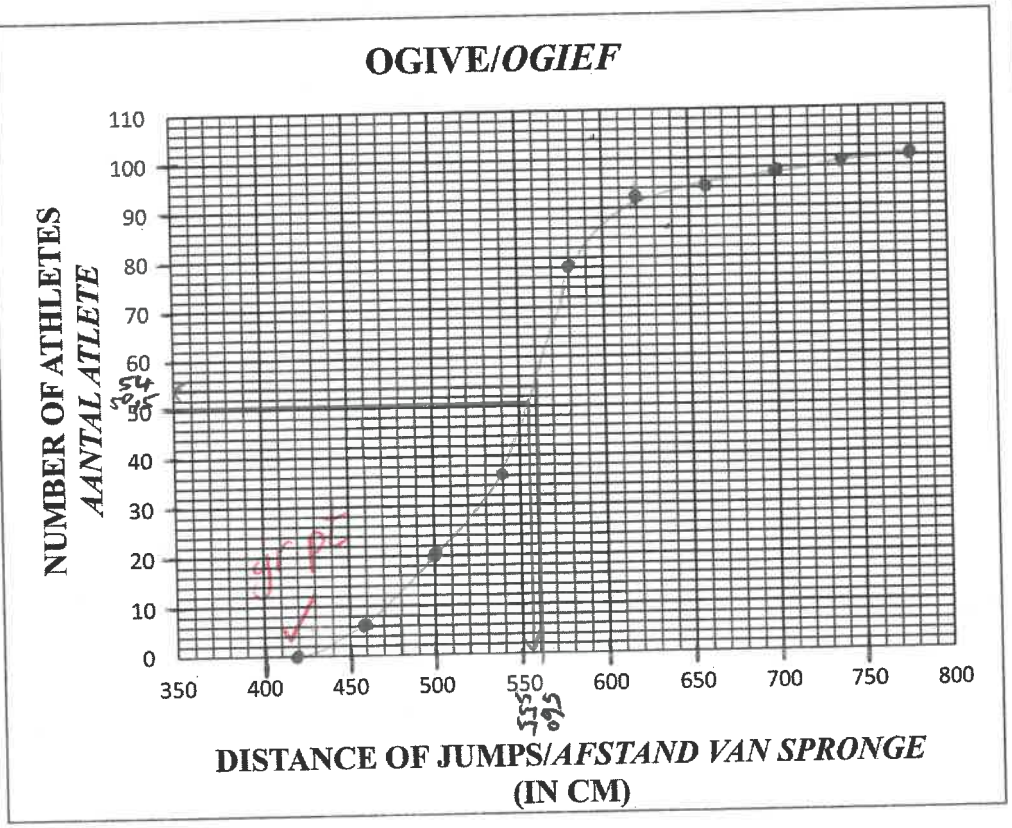
	Solution/*Oplossing*	Marks/ *Punte*

1.1

Distance of Jumps *Afstand van Sponge* (in cm)	Number of athletes *Aantal atlete*	Cumulative Frequency *Kumulatiewe Frekwensie*
$420 < d \leq 460$	6	6
$460 < d \leq 500$	14	20
$500 < d \leq 540$	16	36
$540 < d \leq 580$	42	78
$580 < d \leq 620$	14	92
$620 < d \leq 660$	2	94
$660 < d \leq 700$	3	97
$700 < d \leq 740$	2	99
$740 < d \leq 780$	1	100

✓ ✓ 2 (2)

1.2



✓ upper end of intervals
✓ smooth curve
✓ (780 ; 100)
4 (4)

1.3 $M = T_{\frac{1}{2}(1+100)} = T_{50,5} = 555$ cm ✓ ✓ 2 (2)

1.4 ≤ 560 cm $= 54$ ✓
$\therefore > 560 = 100 - 54 = 46$ athletes
$\therefore \frac{46}{100} \times 100 = 46\%$ ✓
2 (2)

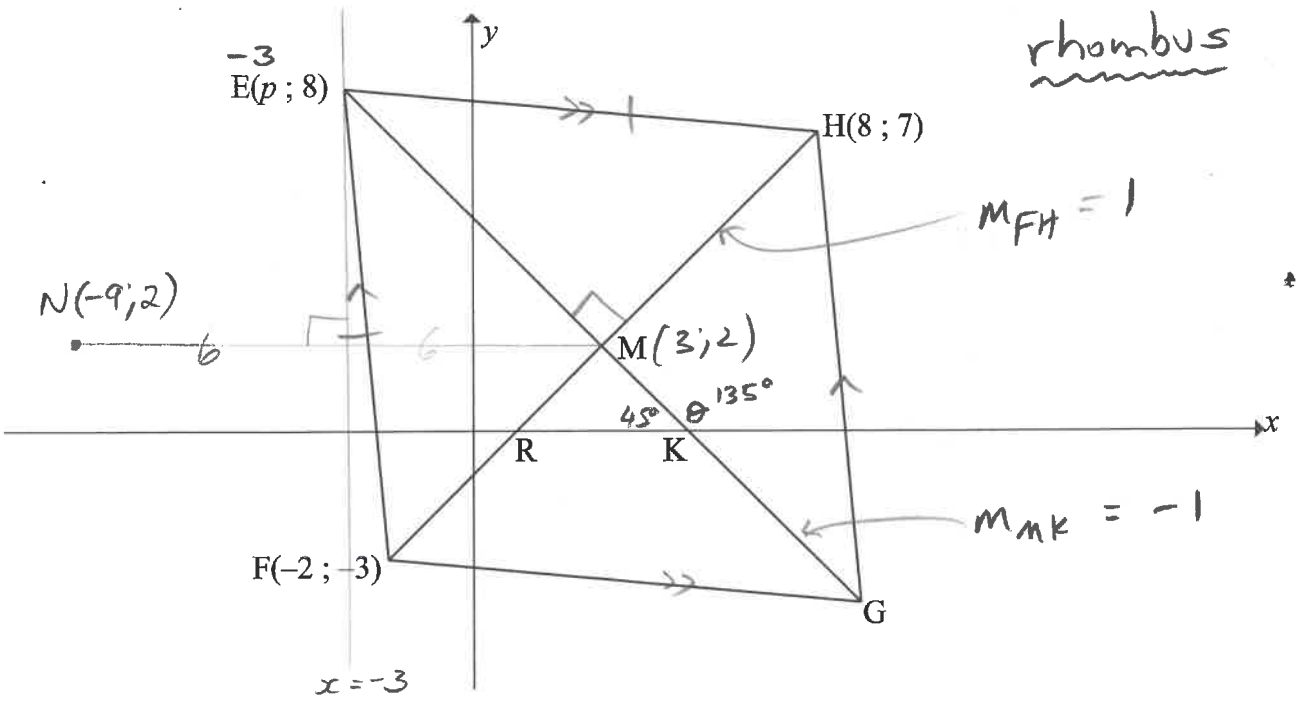
[10]

QUESTION 2/*VRAAG 2*

Long jumper / *Verspringer*	1	2	3	4	5	6
x*: Hours practised / *Ure geoefen	4,5	2	3,5	4	8	3
***y*: Distance jumped / *Afstand gespring* (cm)**	650	420	580	490	780	525

	Solution/*Oplossing*	**Marks/ *Punte***
2.1	$A = 336,70$ ✓ $B = 56,99$ ✓ $\therefore y = 336,70 + 56,99x$ ✓	3 (3)
2.2	$y = 336,70 + 56,99(5,4)$ ✓ $= 644,45$ cm ✓	2 (2)
2.3	$r = 0,92$ ✓ : strong ✓ positive correlation ∴ very valid.	2 (2)
2.4.1	$\bar{x}$: decrease by 13 cm ✓	1 (1)
2.4.2	R : remain the same. ✓	1 (1)
2.4.3	σ : remain the same. ✓	1 (1)
		[10]

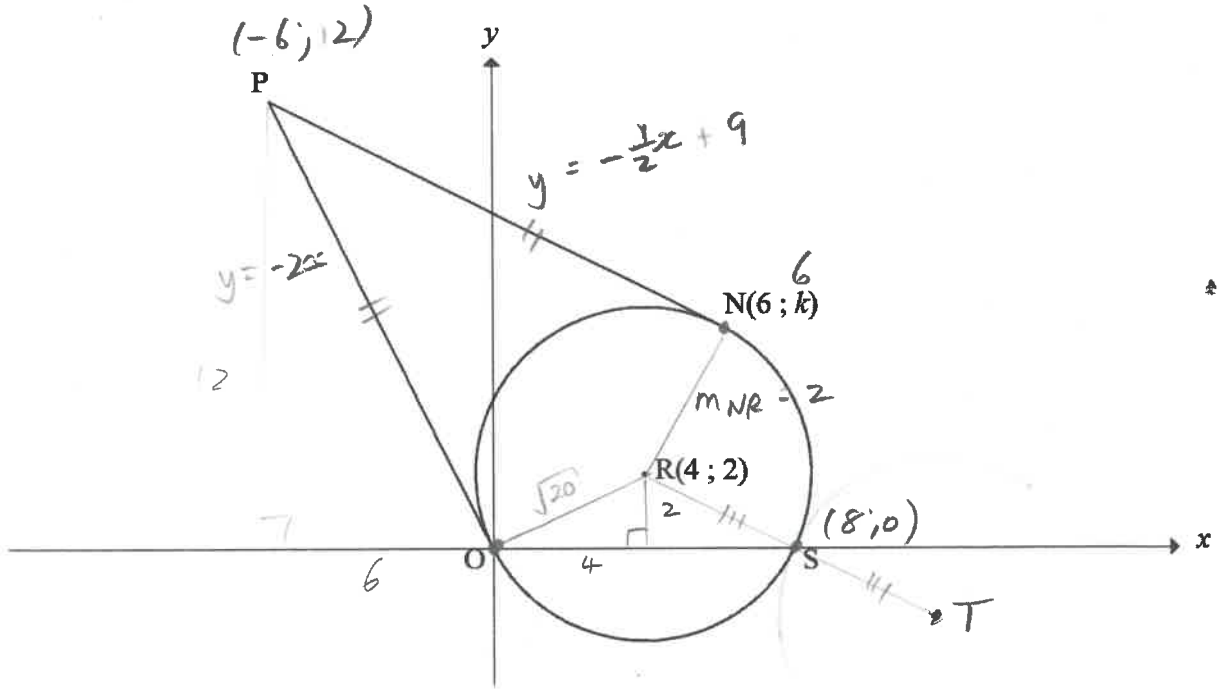
QUESTION 3/*VRAAG 3*



	Solution/*Oplossing*	Marks/ *Punte*
3.1.1	F(−2; −3) H(8; 7) $x_M = \frac{-2+8}{2}$ $y_M = \frac{-3+7}{2}$ $= 3$ ✓✓ $= 2$ ∴ M(3; 2)	2 (2)
3.1.2	F(−2; −3) H(8; 7) $m_{FH} = \frac{7-(-3)}{8-(-2)}$ ✓ $= 1$ ✓	2 (2)
3.1.3	$m_{MK} = -1$ ✓S diags rhomb ⊥ $\tan\theta = -1$ ✓ ref ∠ = 45° tan − in II: $\theta = 135°$ ✓ ∴ $M\hat{K}R = 45°$ ✓S ∠'s on str line = 180°	4 (4)

	Solution/*Oplossing*	**Marks/ *Punte***
3.2	$E(p;8)$ $M(3;2)$ $m_{ME} = \frac{8-2}{p-3}$ $-1 = \frac{6}{p-3}$ diags $\perp$ $-1(p-3) = 6$ $p = -3$ ✓✓✓✓	4 (4)
3.3	$E(-3;8) \xrightarrow[\downarrow 1]{\rightarrow 11} H(8;7)$ $F(-2;-3) \xrightarrow[\downarrow 7]{\rightarrow 11} G(9;-4)$ $EH \parallel FG$ ✓ ✓	2 (2)
3.4	$N(-9;2)$ ✓ $M(3;2)$ $MN = 6 + 6$ $= 12$ u ✓✓ CR if $x_N < -3$ • ans only 2/3	3 (3)
		[17]

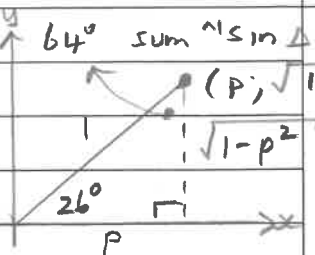
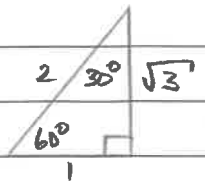
QUESTION 4/*VRAAG 4*



	Solution/*Oplossing* $(x-4)^2+(y-2)^2=r^2$ sub $O(0;0)$	**Marks/ *Punte***
4.1	$(0-4)^2+(0-2)^2=r^2$ ✓ $20=r^2$ ✓ $(x-4)^2+(y-2)^2=20$ ✓	3 (3)
4.2	$(6-4)^2+(k-2)^2=20$ ✓sub $N(6;k)$ $(k-2)^2=16$ ✓ $k^2-4k-12=0$ $k-2=\pm 4$ ✓ $(k-6)(k+2)=0$ $k=2\pm 4$ $k=6$ or ~~-2~~ reject ✓ans with selection	4 (4)
4.3	$m_{NR}=\frac{6-2}{6-4}$ $N(6;6)$ $R(4;2)$ $=2$ ✓ $\therefore m_{NP}=-\frac{1}{2}$ ✓ tan ⊥ rad $y=-\frac{1}{2}x+c$ Sub $N(6;6)$ $6=-\frac{1}{2}(6)+c$ ✓ $9=c$ ✓ $\therefore y=-\frac{1}{2}x+9$ ✓	5 (5)

	Solution/*Oplossing*	Marks/ *Punte*
4.4.1	$y = -2x$ $\quad y = -\frac{1}{2}x + 9$ ← CA if $m < 0$ only $-2x = -\frac{1}{2}x + 9$ ✓ $-\frac{3}{2}x = 9$ $x = -6$ $\therefore y = -2(-6)$ $= 12$ $\therefore P(-6;12)$ ✓ ✓	3 (3)
4.4.2	$OR = RN = \sqrt{20}$ ✓ radii, 4.1. $OP = \sqrt{(12-0)^2 + (-6-0)^2}$ ✓ $P(-6;12)$ $= \sqrt{180}$]✓ $= NP$ tan's from ext common pt = $\therefore$ Perimeter $= 2 \times \sqrt{180} + 2 \times \sqrt{20}$ $= 16\sqrt{5}$]✓ either $= 35,78$	4 (4)
4.5	$S(8;0)$ ✓ $\quad R(4;2)$ $\frac{x_T + 4}{2} = 8$ $\quad \frac{y_T + 2}{2} = 0$ $x_T = 12$ $\quad y_T = -2$ ✓ $\therefore T(12;-2)$ ans only 4/4	4 (4)
		[23]

QUESTION 5/*VRAAG 5*

	Solution/*Oplossing*	Marks/ *Punte*
5.1.1	$\cos 26^\circ = \frac{p}{1}$ $\frac{x}{r}$ $y = \sqrt{1-p^2}$ Pythag $\sin 26^\circ = \frac{\sqrt{1-p^2}}{1}$ $= \sqrt{1-p^2}$ ✓✓ 	2 (2)
5.1.2	$\tan 154^\circ = \tan(180^\circ - 26^\circ)$ $= -\tan 26^\circ$ ✓ ✓ -sign $= -\frac{\sqrt{1-p^2}}{p}$ ✓ $\frac{y}{x}$	3 (3)
5.1.3	$\sin 2x = 2\sin x \cos x$ $13^\circ < 26^\circ$ $P \to L$ $\sin 26^\circ = 2\sin 13^\circ \cos 13^\circ$ ✓ $x = 13^\circ$ $\sqrt{1-p^2} = 2\sin 13^\circ \cos 13^\circ$ $\therefore \frac{\sqrt{1-p^2}}{2} = \sin 13^\circ \cos 13^\circ$ ✓	2 (2)
5.2.1	• $\cos(-\theta)$ $= \cos\theta$ • $\tan(180^\circ + \theta)$ $= \tan\theta$ • $\cos(90^\circ + \theta)$ $= -\sin\theta$ $\therefore \frac{(\cos\theta)(\tan\theta)}{2(-\sin\theta)}$ ✓ $= \frac{\cos\theta \cdot \frac{\sin\theta}{\cos\theta}}{-2\sin\theta}$ ✓ $= -\frac{1}{2}$ ✓	5 (5)
5.2.2	• $\cos 105^\circ$ $= \cos(90^\circ + 15^\circ)$ $= -\sin 15^\circ$ ✓  $\therefore 1 + 2(-\sin 15^\circ)\sin 15^\circ$ $= 1 - 2\sin^2 15^\circ$ ✓ $= \cos 30^\circ$ ✓ $= \frac{\sqrt{3}}{2}$ ✓	4 (4)

	Solution/*Oplossing*	**Marks/ *Punte***
5.3.1	LHS $= \frac{1-(1-2\sin^2 x)-\sin x}{2\sin x\cos x-\cos x}$ ✓ ✓ $= \frac{1-1+2\sin^2 x-\sin x}{\cos x(2\sin x-1)}$ ✓ $= \frac{\sin x(2\sin x-1)}{\cos x(2\sin x-1)}$ ✓both MCF $= \tan x$ $=$ RHS	4 (4)
5.3.2	$\sin 2x-\cos x=0$ ✓den = 0 see w/o → $x = 90°; -90°; 30°$ or $150°$ ✓two values ✓two values	3 (3)
5.4	$\sin^2 x+2\sin x\cos x-3\cos^2 x=0$ ✓std form $(\sin x-\cos x)(\sin x+3\cos x)=0$ ✓factors $\therefore \sin x-\cos x=0$ or $\sin x+3\cos x=0$ $\frac{\sin x}{\cos x}-\frac{\cos x}{\cos x}=0$ $\frac{\sin x}{\cos x}+\frac{3\cos x}{\cos x}=0$ ✓÷cos $\tan x-1=0$ $\tan x+3=0$ $\tan x=1$ ✓ $\tan x=-3$ ✓ ref $\angle = 45°$ ref $\angle = 71,56...°$ tan + in tan − in I: $x=45°+k.180°; k\in\mathbb{Z}$ II: $x=108,43°+k.180°; k\in\mathbb{Z}$ ✓ ✓	7 (7)
		[30]

5.3.2 ID not valid when :

- $\sin 2x - \cos x = 0$

  see ID

  $\cos x(2\sin x - 1) = 0$

  $\cos x = 0$ or $\sin x = \frac{1}{2}$

  $x = 90° + k\,180°;\ k \in \mathbb{Z}$

  $\text{Ref}^{\text{n}} = 30°$

  Sin + ve

  I: $x = 30° + k\,360°;\ k \in \mathbb{Z}$

  or

  II: $x = 150° + k.360°;\ k \in \mathbb{Z}$

or

- $\tan x = UD$

  $\frac{\sin x}{\cos x} = UD$

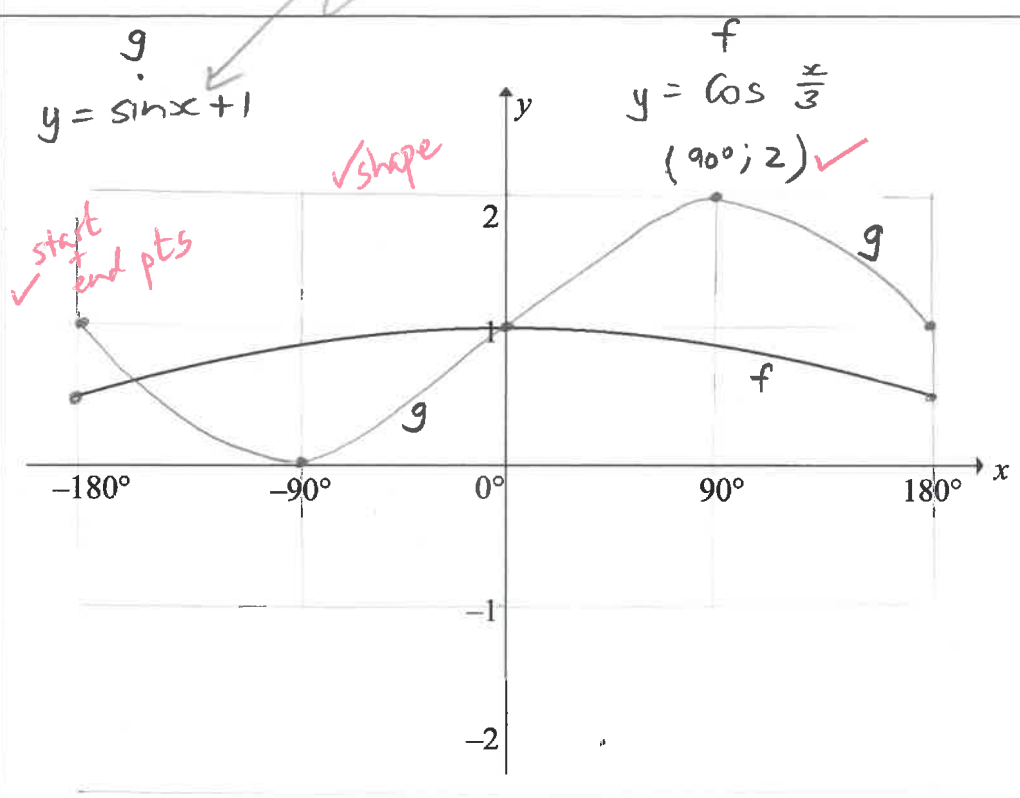
  $\cos x = 0$

  done

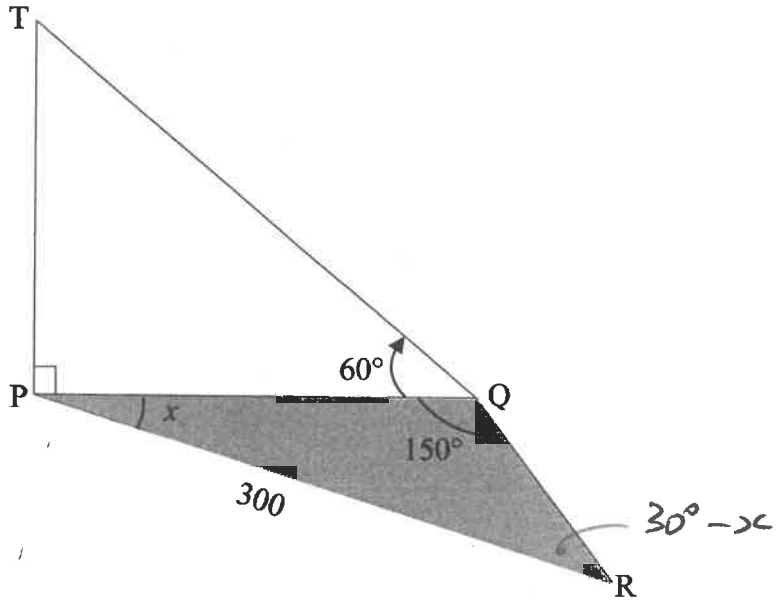
$\therefore$ $x = 90°;\ -90°;\ 30°$ or $150°$

QUESTION 6/*VRAAG 6*

NB : $y = \sin(x) + 1$ on table mode

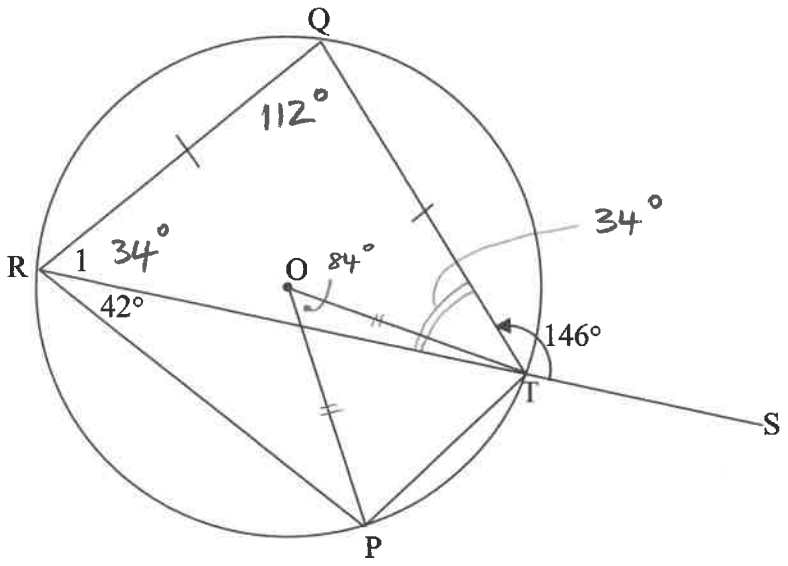
	Solution/*Oplossing*	Marks/ *Punte*
6.1		3 (3)
6.2.1	Period$_f$ = $\frac{360°}{\frac{1}{3}}$ = 1080° ✓	1 (1)
6.2.2	$y \in [-3; -1]$ ✓interval ✓notn -only if int is correct	2 (2)
6.3	Max $g(x) - h(x)$ = 2 − (−2) ✓ = 4 U ✓ ans only 2/2	2 (2)
6.4	$y_f \cdot g'$ + $x \in (-90°; 90°)$ ✓✓ + + + + + y_f − 0 + 0 − g' −90° 90° − 0 ⊕ 0 −	2 (2)
6.5	g moves horizontally 15° to the right ✓ and then vertically 1 unit downwards ✓	2 (2)
		[12]

QUESTION 7/*VRAAG 7*



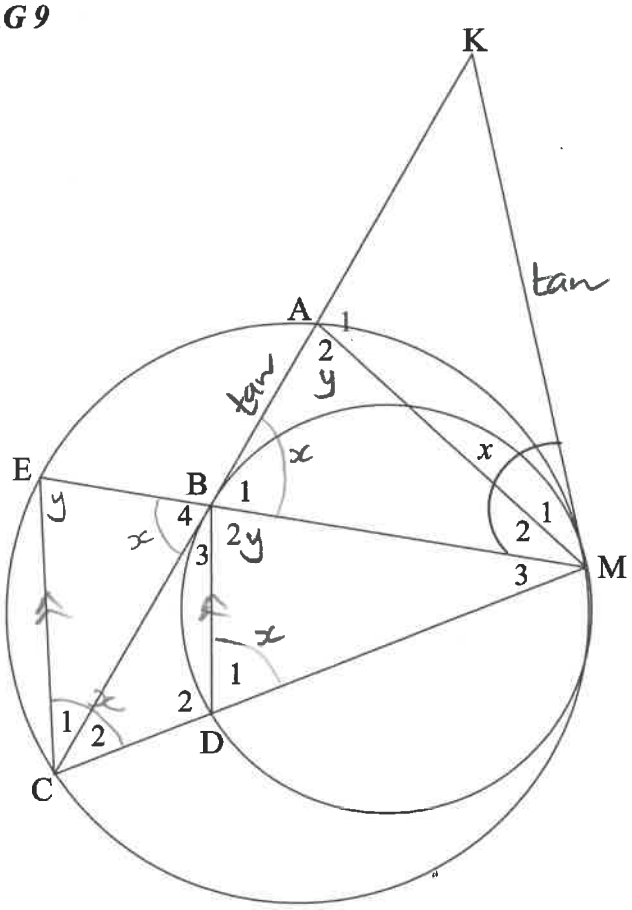
	Solution/*Oplossing*	**Marks/ *Punte***
7.1	$\hat{R} = 30° - x$ ✓S sum ∠'s in Δ = 180°	1 (1)
7.2	$\frac{PQ}{\sin(30°-x)} = \frac{300}{\sin 150°}$ ✓ $\frac{PQ}{\sin(30°-x)} = 600$ ✓600 $PQ = 600\sin(30° - x)$ ✓	3 (3)
7.3	$\frac{TP}{PQ} = \tan 60°$ ✓tan 60° $TP = PQ \tan 60°$ $= (600\sin(30° - x))\sqrt{3}$ ✓sub PQ $= \sqrt{3}\,600(\sin 30°\cos x - \cos 30°\sin x)$ ✓ $= \sqrt{3}\,600(\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x)$ $= \sqrt{3}\,600 \cdot \frac{1}{2}(\cos x - \sqrt{3}\sin x)$ ✓ $= \sqrt{3}\,300(\cos x - \sqrt{3}\sin x)$	expand 4 (4)
		[8]

QUESTION 8/*VRAAG 8*



	Solution/*Oplossing*	**Marks/ *Punte***
8.1	$P\hat{O}T = 84°$ ✓S ✓R ^@centre = 2x ^@ circum	2 (2)
8.2	$Q\hat{T}R = 34°$ ✓SR ^'s on str line = 180° $\therefore \hat{R}_1 = 34°$ ✓SR ^'s opp = sides	2 (2)
8.3	$\hat{Q} = 112°$ ✓SR ext ^ Δ $\therefore R\hat{P}T = 68°$ ✓S ✓R opp ^'s cyclic quad = 180°	3 (3)
		[7]

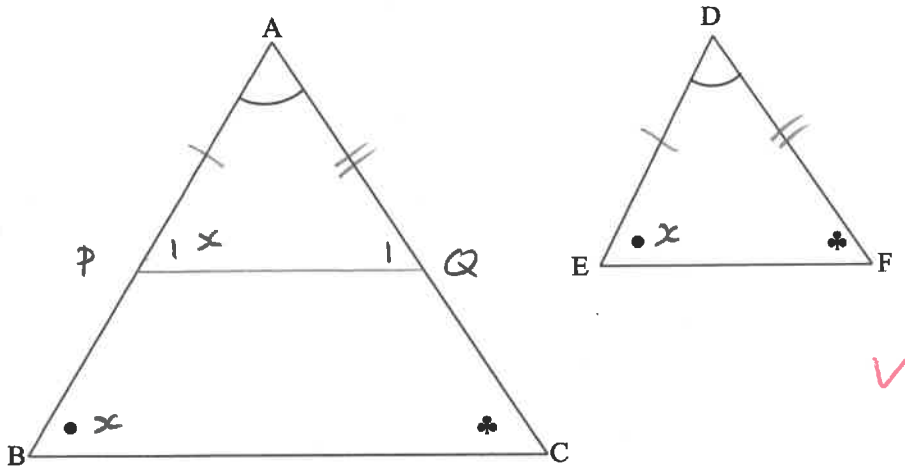
QUESTION 9/*VRAAG 9*



	Solution/*Oplossing*	Marks/ *Punte*
9.1	$KB = KM$ tan's from ext common pt =	5
	$\hat{B}_1 = x$ ✓SR ^'s opp = sides	
	$\hat{B}_4 = x$ ✓SR vert opp ^'s =	
	$\hat{D}_1 = x$ ✓S ✓R ^ tan chord	
	$\hat{C}_1 + \hat{C}_2 = x$ ✓SR ^ tan chord	(5)
9.2.1	$\hat{D}_1 = \hat{C}_1 + \hat{C}_2$ ✓S both = x	2
	$\therefore BD \parallel EC$ ✓R corr ^'s =	
		(2)
9.2.2	Let $\hat{B}_2 = y$	3
	$\therefore \hat{E} = y$ ✓SR corr ^'s =, $EC \parallel BD$	
	$\therefore \hat{A}_2 = y$ ✓S ✓R ^'s in same ⊙ segm =	
	$\therefore \hat{A}_2 = \hat{B}_2$ both = y	(3)
9.2.3	$\frac{ME}{MB} = \frac{MC}{MD}$ ✓S ✓R line $\parallel$ 1 side of Δ	2
	$\therefore ME \cdot MD = MC \cdot MB$	
		(2)
		[12]

QUESTION 10/*VRAAG 10*

10.1



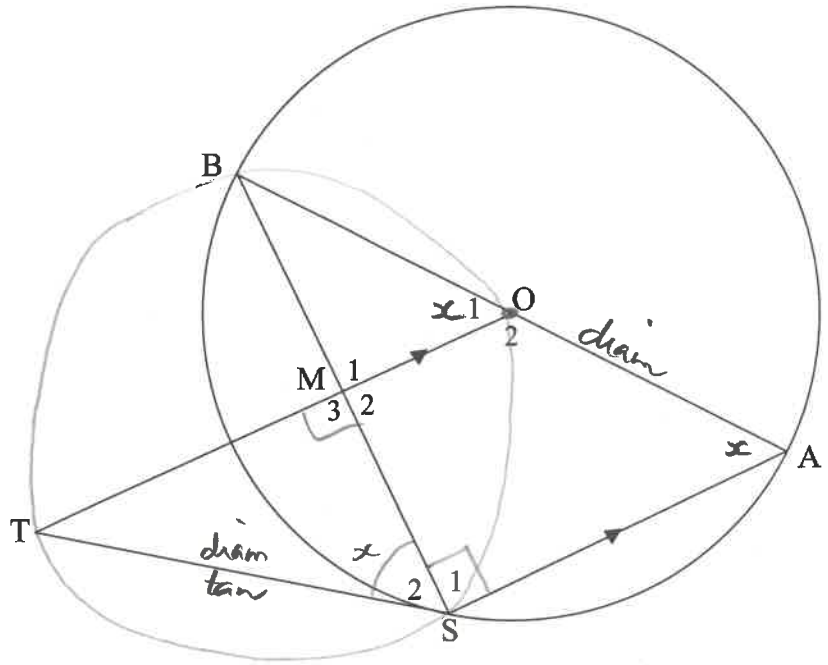
✓ constr I II MUST BE SHOWN

• no constr b/d 0/6

	Solution/*Oplossing*	Marks/ *Punte*
10.1	In $\triangle$'s APQ, DEF	6
	1. AP = DE constr	
	2. AQ = DF constr	
	3. $\hat{A} = \hat{D}$ given	
	$\therefore \triangle APQ \equiv \triangle DEF$ SAS ✓SR	
	Let $\hat{B} = x$	
	$\therefore \hat{E} = x$ given	
	$\therefore \hat{P}_1 = x$ $\triangle APQ \equiv \triangle DEF$	
	$\therefore \hat{B} = \hat{P}_1$ ✓S both = x	
	$\therefore$ PQ ∥ BC ✓S ✓R corr ∠s =	
	$\therefore \frac{AP}{AB} = \frac{AQ}{AC}$ SR ✓ line ∥ side of $\triangle$	(6)

$\therefore \frac{DE}{AB} = \frac{DF}{AC}$ AP = DE AQ = DF

10.2



	Solution/*Oplossing* let $\hat{S}_2 = x$	Marks/ *Punte*
10.2.1	$\hat{O}_1 = x$ ✓S ✓R ∠'s in same ⊙ segm =	4
	$\therefore \hat{A} = x$ ✓SR corr ∠'s =, AS ∥ OM	
	$\therefore \hat{S}_2 = \hat{A}$ both $= x$	
	$\therefore$ TS is tan ✓R conv ∠ tan chord	(4)
10.2.2	$\hat{S}_1 = 90°$ ✓S ✓R ∠ in semi ⊙ $= 90°$	5
	$\therefore \hat{M}_3 = 90°$ ✓S ✓R alt ∠'s =, AS ∥ OM	
	$\therefore$ TS is diam ✓R conv ∠ in semi ⊙ $= 90°$	
	to ⊙ TMS	
		(5)
10.2.3	In Δ's A B S_1, S_2 T M_3	3
	1. $\hat{A} = \hat{S}_2$ ✓S both $= x$	
	2. $\hat{S}_1 = \hat{M}_3$ ✓S both $= 90°$	
	3. $\hat{B} = \hat{T}$ ∠'s in same ⊙ segm =	
	$\therefore \Delta$ ABS ||| Δ STM A A A ✓R	(3)
10.2.4	$\frac{AS}{SM} = \frac{BS}{TM}$ ✓S Δ ABS ||| Δ STM	3
	$\therefore$ AS . TM = SM . BS	
	BM = SM ✓R line from centre ⊙ ⊥ to chord	
	$\therefore$ BS = 2 . SM ✓S	
	$\therefore$ AS . MT = SM . 2SM	(3)
	$= 2SM^2$	[21]